Fermi National Accelerator Laboratory

FERMILAB-Conf-91/56-A
February 1991

ULTRAHIGH-ENERGY PARTICLES FROM COSMIC STRINGS†

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ABSTRACT

The idea of production of ultrahigh-energy particles in the present universe due to annihilation or collapse of topological defects is discussed. Topological defects, e.g., monopoles, cosmic strings, domain walls, etc., formed in symmetry-breaking phase transitions in the early universe, can survive till today owing to their topological stability. However, under certain circumstances, topological defects may be physically destroyed. This may happen, for example, when monopoles annihilate with antimonopoles, or when closed loops of cosmic strings or closed domain walls collapse. When topological defects are destroyed, the energy contained in the defects can be released in the form of massive gauge- and higgs bosons of the underlying spontaneously broken gauge theory. Subsequent decay of these massive particles can give rise to energetic particles ranging upto an energy on the order of the mass of the original particles released from the defects. This may give us a 'natural' mechanism of production of extremely energetic cosmic ray particles in the universe today, without the need for any acceleration mechanism. To illustrate this idea, I describe in detail the calculation of the expected ultrahigh-energy proton spectrum due to a specific process which involves collapse or multiple self-intersections of a class of closed cosmic string loops formed in a phase transition at a grand unification energy scale $\sim 10^{15} GeV$. I discuss the possibility that some of the highest-energy cosmic ray particles are of this origin. By comparing with the observational results on the ultrahigh-energy cosmic rays, we derive an upper limit to the average fraction of the total energy in all "primary" cosmic string loops that may be released in the form of particles due to collapse or multiple self-intersections of these loops. Characteristic features of topological defect-induced cosmic ray spectrum include 'recovery' of the spectrum after the Greisen-Zatsepin-Kuz'min 'cutoff', continuation of the spectrum essentially to a grand unification energy scale $\sim 10^{24} eV$, and presence of only 'fundamental' particles such as protons, neutrons, etc., and their antiparticles, but *no* nuclei such as α's or Fe's in the spectrum. It is emphasized that discovery of cosmic ray particles significantly above $10^{21} eV$ in future ultrahigh-energy cosmic ray detectors with large area coverage may provide us with one of the few signatures of Grand Unified Theories in general and existence of topological defects in particular.

† Invited talk given at the ICRR International Symposium on *Astrophysical Aspects of the Most Energetic Cosmic Rays*, held in Kofu, Japan, 26–29 November 1990. To appear in the Proceedings to be published by World Scientific, Singapore.

1. Introduction

In this talk I discuss the possibility that topological defects,[1,2] in particular cosmic strings, predicted in some Grand Unified Theories[3] (GUTs) may, under certain circumstances, be sources of extremely energetic particles in the universe today. I shall also discuss the possibility that some of the highest energy cosmic ray particles are of this origin. Primarily, however, my aim here will be to use the observed ultrahigh-energy (UHE) (i.e., energy greater than, say, $10^{18}eV$) cosmic ray (CR) spectrum to put constraints on the properties of the topological defect under consideration from the requirement that the UHE particle flux produced by the defects not exceed the observed UHE CR flux at any energy. As we shall see below, the maximum energy of the particles produced from the defects can go up to some typical GUT energy scale $\sim 10^{15}GeV$. We will therefore also have interesting prediction about possible UHE particle flux at energies much beyond the highest currently observed CR energies ($\sim 10^{20}eV$). Although the predicted flux at these extremely high energies is very small, one nevertheless hopes that some of the ongoing and planned future CR experiments may have large enough area coverage to be able to detect these particles, if they exist.

One can think of a variety of different UHE particle production processes involving various different kinds of topological defects. In this talk I shall only consider UHE particle production due to one specific process[4] involving one specific kind of topological defect, namely, cosmic strings. But before I do that let me first try to motivate and explain the general idea behind the mechanism of production of UHE particles from topological defects. Throughout this talk I shall use, unless otherwise specified explicitly, natural units with $c = \hbar = M_{Pl}\sqrt{G} = k_B = 1$, where G is Newton's constant, M_{Pl} is Planck mass, and k_B is Boltzmann constant. Also, all numerical results will refer to a cosmological model which corresponds to a "flat" ($\Omega_0 = 1$) universe with a Hubble constant in the present epoch $H_0 = 75\,Km.s^{-1}.Mpc^{-1}$, and the temperature of the cosmic microwave background radiation, $T_{CMBR} = 2.75^o K$. In this model, the age of the universe is $t_0 = 2.74 \times 10^{17} sec$, the time of equal matter- and radiation- energy density is $t_{eq} \simeq 1.4 \times 10^{11} sec$ which corresponds to a redshift $z_{eq} \simeq 1.57 \times 10^4$.

2.Production of UHE Particles from Topological Defects: Motivation and the General Idea

One of the motivations for considering topological defects as the sources of UHE particles is that in this case particles can be 'naturally' produced at UHE energies without any acceleration mechanism. The primary problem in trying to understand the origin of UHE CR lies in the fact that it is very hard to come up with a suitable mechanism by which particles (protons and/or iron nuclei, believed to be the primary constituents of UHE CRs) could be accelerated to such high energies. There exist in literature various scenarios[5] in which particles are accelerated in certain special astrophysical environments such as in supernova shocks, in pulsar magnetospheres, in galactic wind termination shocks, in active galactic nuclei, in hot spots of radio galaxies, and a host of other suggested astrophysical sites. Some of these acceleration mechanisms have been discussed at length by various speakers at this meeting.[6] Although it has been claimed that some of these mechanisms are able to accelerate particles to the highest observed CR energies, it is, I think, fair to say that, at this time, the problem is far from being regarded as completely solved. In any case, one can, I think, certainly say that it will be extremely difficult, if not impossible, for any of the currently known astrophysical acceleration mechanisms to explain the existence of CR particles with energies significantly higher than say $10^{21} eV$, should such particles be detected in future.

Given this circumstance, I would like to explore here a different possibility. One asks the question: Is it possible that UHE CR particles above some energy may actually have a "fundamental" origin, in the sense that they are *not* the result of any acceleration mechanism at all but are simply the decay products of some sufficiently massive particles which were created in some early epoch in the history of the universe and which somehow survived until recent epochs? The decays must take place in recent epochs because the particles produced from decays of the massive particles in early epochs would thermalize and essentially lose all energy during their propagation through the cosmic medium, and so would not survive as UHE particles today. As for the existence of the massive particles themselves, given the enormously high energies we are concerned with, it is natural to seek the answer to the above question within the context of GUTs. In any GUT, the particle spectrum typically contains supermassive gauge- and higgs bosons (which we shall hereafter generically refer to as "X-particles"), with masses given by a typical GUT energy scale which can indeed be large $\sim 10^{15} GeV$. But the problem here is that these particles also normally have extremely short lifetimes. A typical superheavy gauge boson, for example, decays with a decay rate[7] (for $T \lesssim m_X$, T being the temperature of the universe), $\Gamma_{decay} \sim \alpha_{GUT} m_X$, where $\alpha_{GUT} \approx \frac{1}{45}$ is a typical gauge coupling constant in GUT, and m_X is the mass of the gauge boson. For $m_X \approx 10^{15} GeV$, one gets a lifetime $\tau \sim \Gamma_{decay}^{-1} \approx 3 \times 10^{-38} sec$. These particles therefore disappear quickly when $T \ll m_X$, because the reverse reactions producing these particles are then blocked. This happens rather early in the universe and one would therefore conclude that essentially none of these massive particles would survive till recent epochs.† The above argument is, however, valid only for *free* X-particles. There is, in fact, an important circumstance in which the massive gauge- and higgs bosons are *not* free, and are, in a sense, *prevented from decaying.* This happens *when topological defects are present.* In the next section we will briefly discuss the nature of topological defects and their formation during spontaneous symmetry-breaking phase transitions in the early universe. For the present, the important point to note about topological defects is that they essentially 'trap' the

† It should perhaps be mentioned that there are particle physics models, e.g., a recent one (based on Superstring theory) given by Ellis *et al*,[8] in which there exist massive long lived (meta)stable particles, the so-called "cryptons", which have a mass $\sim 10^{12} GeV$; we will not consider these particles here although they may have important implications.

gauge- and higgs boson fields in topologically nontrivial configurations which under normal dynamical evolution cannot by themselves 'unwind' and are therefore stable due to topological reasons. Well-known examples of topological defects are magnetic monopoles, cosmic strings, and domain walls. Most GUT models do in fact predict one or more kinds of topological defects. (Magnetic monopoles are predicted in all GUT models). Because of their topological stability the defects can survive long times (on cosmological time scales), *until and unless physically destroyed.* The physical destruction of the defects is of course possible and is in fact crucial in our scenario. This may happen, for example, when a magnetic monopole annihilates with an antimonopole, or, say, when a closed loop of cosmic string collapses; one can think of other processes which also lead to destruction of cosmic strings, domain walls, etc. When topological defects are physically destroyed, the massive gauge- and higgs bosons trapped inside the defects are released— the topologically nontrivial configuration of the gauge-higgs field system can then unwind and dissociate, so to say, into quanta of gauge- and higgs fields which 'constitute' the string. These massive quanta then decay freely and give rise to UHE particles in a way I shall discuss in more detail later. So the general scenario we are envisaging here is the following: Topologically stable defects are formed during (possibly a GUT-) symmetry breaking phase transition in the early universe. Some of these defects survive till recent epochs and then destroy themselves, releasing their energy primarily in the form of massive particles of the underlying spontaneously broken gauge theory. These massive particles then decay and produce UHE particles. The existence of topological defects thus provides us with a possible mechanism of production of UHE particles without the need for any acceleration mechanism.

At this point I should perhaps emphasize that it will be too simplistic to expect that topological defects give the sole contribution to the entire UHE CR spectrum— after all, there are other well-known astrophysical processes which may be quite adequate for accelerating CRs up to certain maximum energy. Where topological defects may, however, be relevant is probably only in the highest energy part of the UHE CR spectrum, in particular, for energies beyond $\sim 10^{20} eV$, where currently known astrophysical acceleration mechanisms encounter great difficulties. In any case, if the general idea behind GUT is true, then within the context of the Hot Big Bang model of the early universe, formation of topological defects is predicted in most realistic GUT models,† and, therefore, I think it is an important problem in its own right to study the implications of the process of UHE particle production from these defects, and to see if any observable consequences can be obtained. This, then, is the primary motivation behind the subject of this talk.

Before proceeding further, let me mention here the previous works on this subject of UHE particle production by topological defects. Hill[10] considered UHE particle production due to hadronization of energetic gluons emitted (by classical Lar-

† There are complications arising from the fact that inflation tends to 'inflate away' any topological defects formed previous to any inflationary phase transition. However, the defects can be formed in phase transitions taking place *after* or even *during* the inflation.[9]

mor radiation) from decaying monopolia (monopole-antimonopole bound states). He also considered particle production in the final burst when the monopole and the antimonopole annihilate each other. However, Hill did not give any calculation of the expected contribution, if any, of the monopolia to the UHE cosmic rays in the present universe. Hill, Schramm and Walker[11] considered production of UHE particles from the decay of massive fermions released by closed loops of superconducting cosmic strings[12] when they attain a maximum allowable current[12]. These authors calculated the expected UHE proton- and neutrino spectra in the present epoch, due to this process. Bhattacharjee,[13] and MacGibbon and Brandenberger[14] studied UHE particle production due to decay of massive particles released from 'ordinary' cosmic strings by the so-called "cusp evaporation process". Finally, UHE particle production by collapsing closed cosmic string loops — the main process discussed in this talk — has been discussed recently by Bhattacharjee and Rana.[4] Further discussions of topological defect-induced UHE particle production are given in Bhattacharjee[15] and in Bhattacharjee, Hill and Schramm.[16]

3. Symmetry-breaking Phase Transitions and Topological Defects

In this section we discuss some basic ideas relating to formation of topological defects in symmetry-breaking phase transitions in the early universe. Readers familiar with this subject may wish to skip to Section 4.

3.1. The Basic Idea of Spontaneous Symmetry Breaking

The existence of topological defects is intimately connected with the phenomenon of Spontaneous Symmetry Breaking (SSB). To facilitate our later discussions, let us first briefly recall the basic idea behind the so-called Higgs-Kibble mechanism of SSB in gauge theories.[17] In general, one considers a set of scalar fields (the 'higgs' field) which interact with a set of vector fields (the 'gauge' field) in such a way that the total energy (or to be more precise, the Action) of the system is invariant under some transformations which mathematically form a group called the symmetry group. Let us denote this symmetry group by G. Now, it may so happen that the vacuum state of the system does *not* reflect this invariance; instead, the vacuum state may be invariant only under a restricted set of transformations which form a subgroup (denote it by H) of the original symmetry group G. In such a situation one says that symmetry is spontaneously broken. This usually comes about because of existence, in an SSB situation, of degenerate vacua, i.e., there exist more than one vacua (in general, a whole continuum of them) all of which have the same energy. The degenerate vacua— the so-called 'Higgs vacua' — correspond to existence of degenerate minima of the potential energy of the higgs field. Any transformation belonging to the symmetry group G but not belonging to H takes one vacuum into another. Thus any particular vacuum state chosen by the system will be invariant only under the transformations belonging to the subgroup H and *not* the full symmetry group G— one says that the symmetry group G is spontaneously broken by the vacuum to the group H. Note the crucial role played by the higgs field in the phenomenon of SSB. It is well known, but I state it here for completeness, that the spectrum of physical particles in any spontaneously broken gauge theory (SBGT) has, among other particles, one or more *massive* vector (gauge) bosons and *massive* scalar (higgs) bosons which appear, so to say, 'in the act of symmetry breaking'. Indeed, the existence of these massive gauge- and higgs bosons is regarded as one of the signatures of any SBGT.

3.2. GUTs and Symmetry Restoration at High Temperatures

The concept of SSB is basic to GUTs.[3] As is well known, the different kinds of fundamental interactions observed in nature today (the strong-, the weak- and the electromagnetic interactions) are associated with invariance of the fundamental Action under different symmetry groups. The basic hypothesis in any GUT is that these different symmetry groups are subgroups of a larger "unified" symmetry group. This unified symmetry is, however, not manifest in today's universe— it is spontaneously broken. On the other hand, it is known that broken symmetries may be restored[18] at sufficiently high temperatures. Here by restoration of a symmetry one refers to a situation in which the vacuum state of the system shows the full symmetry of the Action. Connection between symmetry and temperature is well known in physics. An often cited example is of course the ferromagnet: Above the "Curie temperature"(T_c), a ferromagnetic material has no magnetization. There is no preferred spatial direction in this phase and so the physics describing the system is invariant under the full rotational symmetry group. Below T_c, however, the system acquires a net magnetization induced by whatever stray external magnetic field that may happen to be present, and the system loses its invariance under the full rotational symmetry group— the symmetry is broken by the presence now of a preferred spatial direction, namely the direction of magnetization. It is thought that a similar circumstance obtains with regard to symmetries that govern the fundamental interactions in nature. In this case the physical system is our universe itself which, according to the Hot Big Bang model, was hotter (and denser) in the past. So it is possible that in the early universe, when the temperature was sufficiently high, the full unified symmetry of a GUT was restored; the symmetry broke as the universe expanded and cooled through certain critical temperature. In fact, according to GUT, the universe is thought to have passed through at least two, and possibly more, occurrences of symmetry breaking. One of these is associated with the breaking of the unified electroweak symmetry group $SU_2 \times U_1$ and the other is associated with the breaking of the original 'grand unified' symmetry group into the gauge group $SU_3 \times SU_2 \times U_1$, where SU_3 is the so-called 'color' gauge group that describes strong interaction. It is possible that this latter symmetry breaking may have taken place through one or more intermediate steps of symmetry breaking.

3.3. Symmetry-Breaking Phase Transitions and Formation of Topological Defects

Now, what actually happens in the universe in the process of symmetry breaking is that the universe undergoes a phase transition — it passes from a higher symmetry (higher temperature, and hence higher energy) phase to a lower symmetry (lower temperature, and hence lower energy) phase when the temperature falls below a critical temperature. Phase transitions are well known to be accompanied by formation of *defects*. The latter are isolated parts of the system, which continue to remain in the 'old' higher symmetry phase even after the completion of the phase transition when the bulk of the system passes on to the 'new' lower symmetry phase. Again, this comes about because of the presence of multiple degenerate Higgs vacua in the broken-symmetry phase. As the symmetry breaks, one possibility is that different regions of the universe may choose to settle down to different states corresponding to different Higgs vacua of the broken symmetry phase. In fact, as first pointed out by Kibble,[1] because of the presence of 'horizons', regions of the universe separated by more than horizon distance ($\sim ct$, at any time t after the big bang) would be causally disconnected, and there is no reason for the higgs field in causally disconnected regions of the universe to be the same. In this case, certain arrangements of the higgs field in different regions of the universe may, for reasons of continuity of the gauge- and the higgs fields, force the higgs field to vanish, and hence remain invariant under the full unified symmetry, on some points, lines or surfaces depending on the topology of the manifold of the higgs vacua. These points, lines or surfaces, where the unbroken symmetry persists, are called defects. The topologically stable defects correspond to those spatial arrangements of the Higgs vacua which can never evolve, under any continuous dynamical evolution, into a topologically trivial spatial arrangement, namely, an arrangement in which any one particular Higgs vacuum is chosen at all spatial points.

The topological defects represent stable, finite-energy, spatially extended solutions of the classical equations of motion of the gauge- and higgs fields. As already indicated above, depending on the topology of the manifold formed by the Higgs vacua, three fundamentally different kinds of topologically stable defects are possible. These are magnetic monopoles ('point defects'), cosmic strings ('line defects') and domain walls ('surface defects'). The domain walls appear only if a discreet symmetry is broken, whereas magnetic monopoles and cosmic strings arise from breaking of continuous symmetries. The higgs field varies in a continuous manner from zero on the defect points, lines or surfaces, to one of its vacuum values corresponding to the broken symmetry phase outside. The characteristic length scale over which the fields vary is $\sim m_\phi^{-1}$, where m_ϕ is the mass of the higgs field. This gives the defects a finite size or thickness which is, therefore, also $\sim m_\phi^{-1}$. It should also be mentioned that although the gauge field has not featured much in the above discussions, its presence is in fact crucial for making the energy of these defects finite. In fact, as we have indicated earlier in the previous section, the defects can be regarded, in some sense, as 'topological bound states' of the gauge- and higgs fields, and as such can be thought of as 'constituted' of the quanta of these fields.

We will not discuss magnetic monopoles and domain walls any further in this talk. Instead we concern ourselves only with cosmic strings.

4. Cosmic Strings and Production of UHE Particles

4.1. The Basic Nature of Cosmic Strings

Cosmic strings may be formed when any U_1 symmetry group is spontaneously broken. The higgs field in this case is a complex scalar field which is zero on certain (in general curved) lines which we may call 'nodal lines' . Far away from these lines the higgs field is in one of its vacuum configurations, i.e., it takes a value that minimizes the higgs potential. Of course, there are more than one vacua in the theory; in the present case the vacua differ from each other by the value of the phase angle of the complex scalar field. This phase angle changes by 2π as one encircles the nodal line once along any closed circular path centered at any point on the nodal line. The characteristic length scale over which the higgs field varies from zero on the nodal line to one of its vacuum values outside is $\sim m_\phi^{-1}$. Cosmic strings are thus tubular regions of space of thickness $\sim m_\phi^{-1}$ within which the symmetry of the universe is different from the broken symmetry outside. Indeed the exact unbroken symmetry persists on the nodal lines. The 'magnetic' field that corresponds to the local U_1 symmetry (whose breaking gives rise to the formation of the strings) is confined inside the strings and lies parallel to the nodal lines. The thickness (δ) and the energy trapped per unit length of the string (μ) are determined by the masses of the higgs- and the gauge bosons, which, in turn, are determined by the temperature at which the symmetry-breaking phase transition occurs. A grand unified symmetry breaking phase transition at a temperature $\sim 10^{16} GeV$, for example, would give rise to massive gauge- and higgs bosons (the "X"-particles) with a mass m_X of this order. So the cosmic strings formed at such a phase transition (due to spontaneous breaking of a gauged U_1 subgroup of the full grand unified symmetry group) would have $\delta \sim m_X^{-1} \sim 2 \times 10^{-30} cm$, and $\mu \sim m_X^2 \sim 10^{22} g.cm^{-1}$. Thus GUT-scale cosmic strings are extremely thin and heavy objects.

The closest analogues of cosmic strings on laboratory scale physics are the quantized flux tubes observed in the so-called Type II superconductors when the latter are placed in an external magnetic field. Here the analogue of the higgs field is the so-called "Cooper pair"(a bound state of two electrons), which spontaneously breaks the U_1 symmetry that describes electromagnetism. The breaking of electromagnetism inside the superconductor gives the photons an effective mass inside the superconductor. This makes electromagnetism short-ranged inside the superconductor which in turn is responsible for the squeezing of the magnetic field lines in flux tubes. The analogue of the massive gauge field is the 'massive photon' in the superconductor.

For more details on the basic nature of cosmic strings (or, for that matter, other topological defects too), the reader is referred to the very enjoyable original paper of Kibble[1] and the excellent review by Vilenkin.[2]

I mention here in passing that cosmic strings have been extensively studied over the past several years in connection with their possible important role in the problem of formation of galaxies and large scale structure in the universe. I will not touch upon this aspect of cosmic string at all in this talk. For some discussions about the current status of this subject, see Ref.19, for example.

4.2. Formation of Closed Loops of Cosmic String

When cosmic strings are first formed, they have a tangled configuration with the characteristics[20] of a Brownian random walk with a persistence length ξ set by the correlation length of the higgs field at the time of phase transition. Roughly, $\xi \sim m_\phi^{-1}$ at the time of formation of the strings. Some closed loops are formed too, which comprise about 20% of the total string length.[20] The string configuration then evolves due to tension in the string and due to the expansion of the universe. The main thing that happens to the string network is that new closed loops of string form as string segments intersect. The loop[†] formation occurs because intersecting string segments "exchange partners"[1] at the points of intersection and then reconnect the other way. Numerical simulations[21] of string intersections do indeed show the occurrence of this loop formation process. Since no physical interaction process can occur on scales larger than the horizon length, any new loops formed by intersections of strings must be of subhorizon size. There are many ways in which new loops can form. The loops chopped off from *long strings* (a long string is defined to be a string that is *not a closed loop of horizon size or smaller*), due to self-intersections or due to intersections with other strings, will be called the "primary" loops. Needless to say, an already existing loop may also intersect itself and break up into two or more "secondary" loops. In this talk we shall be mainly concerned with the primary loops. The chopping off of primary loops from the long strings govern the way in which the energy density of long strings evolves as the universe expands. In fact, the formation of the primary loops is thought to be the principal mechanism by which long strings get rid of the energy they continually gain due to their stretching[22] caused by the expansion of the universe. Without this loop formation process the energy density of the universe would be dominated by the long strings at very early times, which would be a disaster. The closed loops cannot dominate the energy density of the universe because they lose energy as they oscillate and radiate gravitational waves, or in certain special cases, the loops can collapse and release their energy in the form of other particles.

The average rate of formation of primary loops is given by

$$\frac{dn_f}{dt_f} = \beta \frac{1}{t_f^4}, \tag{1}$$

where n_f is the number density at the time of formation t_f. Eq. 1 describes a situation in which an average number β of subhorizon-sized loops are formed per horizon-sized volume of the universe per Hubble time at any time of formation t_f. The typical length L_f of a loop at any time of formation t_f is given by

$$L_f = \alpha t_f = M_f/\mu, \tag{2}$$

† From now on the word "loop" by itself will mean "*closed* loop".

where $\alpha < 1$ is a numerical constant, M_f is the total energy of the loop at formation, and μ is the energy per unit length of the string, which is determined by the energy-scale at which the string-forming phase transition takes place.

Detail numerical simulations[23] confirm the notion that the process of closed loops chopping off the long string network leads to the so- called "scaling solution"[22-24,2] in which strings constitute a small and constant fraction of the total energy density in the universe at all times. In this scaling solution the energy density in long strings (i.e., length > horizon), ρ_{LS}, is given by $\rho_{LS}(t) = \mu/\xi^2(t)$, where $\xi(t) = \gamma d_H(t)$ is the scale length in the long string network, $d_H(t)$ being the horizon distance at time t, and γ is a *constant* whose value can be determined from the numerical simulations. The net rate of energy loss of long strings to the loops is given, in the scaling solution, by $(d\rho_{LS}/dt)_{to\ loops} = -\kappa\rho_{LS}(t)/\xi(t)$, where κ is another constant, the so-called "loop chopping efficiency" whose value can also be determined from numerical simulation.

Note that the product of the two parameters α and β defined by Eqs. 1 and 2 is related, in the scaling solution for ρ_{LS}, to the constants γ and κ by the relation $\alpha\beta = \kappa t^3 \left(\gamma d_H(t)\right)^{-3}$. For our later calculations we will need the numerical value of this product $\alpha\beta$. For the matter dominated epoch (which is the one of our interest; see below), the numerical simulations of Bennett and Bouchet(BB)[23] give[4] $\alpha\beta \approx 0.57$, while the simulations of Albrecht and Turok(AT)[23] give[4] $(\alpha\beta)_{AT} \approx 0.86$. So it seems that the product $\alpha\beta \lesssim 1$. For definiteness, we shall take in our numerical calculation $\alpha\beta = 0.57$ as given by BB[23]; one can easily scale the final numerical results appropriately if/when a more accurate value becomes available.

I should perhaps mention here that although there is a fair degree of agreement on the numerical value of the *product* $\alpha\beta$ (which reflects the agreement on the "scaling solution" for long string energy density) among the results of the different numerical simulations mentioned above, the situation is remarkably different when one considers the individual values of α and β separately— in fact, there are large quantitative and hence qualitative differences. The BB simulation finds that most of the primary loops are very small compared to the horizon length, i.e., $\alpha \ll 1$, whereas the AT simulation finds most of the primary loops to be of the order of horizon size, i.e., $\alpha \lesssim 1$. The numerical simulation done by Allen and Shellard(AS)[23] gives results which are in general agreement with those of BB. So it seems to be the case that the primary loops chopped off from long strings are indeed very small compared to the horizon size at the time of their formation. The small size of the primary loops in the BB and AS simulations is attributed to their proper accounting of the presence of a great deal of small-scale structures on the long strings. These small-scale structures are thought to arise due to 'kinks' left behind on the strings everytime loops chop off the long strings. The AT simulation code somehow seems to miss these small-scale structures in a way that remains to be sorted out completely.

On the other hand, if it is true that the loops chopped off from the long strings are indeed much smaller than the horizon size, then one would expect that the time scale of formation of these loops would be much smaller than the Hubble time. So if chopping off of the primary loops from the long strings is responsible for the scaling solution for the energy density of the long strings, then it is somewhat of a puzzle at this time as to why the time scale that appears in the scaling solution and in the loop formation rate in Eq. 1 is the Hubble time at the time of formation of the loops, and *not* the much shorter time scale of formation of the loops. While a clear resolution of this puzzle has yet to come, the preliminary suggestion[25-27] seems to be that in some not yet well understood way, the loops form due to intersections of distant regions of long strings (which occurs on the horizon length scale and hence on a Hubble time scale), but the sizes of the loops formed are governed by the scale of small-scale structure (due to kinks) on the long strings. Recent works[25,27] indicate that the typical size of the primary loops is determined by the time scale of decay of the kinks due to gravitational radiation from these kinks. This gives a characteristic size of the primary loops chopped off from the long strings at any time t, of order $\Gamma G\mu t$, so that $\alpha \sim \Gamma G\mu$ with $\Gamma \sim 50$.

4.3. Collapse and Multiple Self-Intersections of Cosmic String Loops and Release of Massive X-Particles

Now, after its formation a closed loop will execute complicated motion due to the tension ($= \mu$) in the string. For subhorizon-sized loops the expansion of the universe can be neglected[22] and the motion of the loop can be described (with a suitable 'gauge'choice[28]) by the "wave equation" $\ddot{\mathbf{X}}(\sigma,t) = \mathbf{X}''(\sigma,t)$, together with the constraint equations $\dot{\mathbf{X}}\cdot\mathbf{X}' = 0$, $\dot{\mathbf{X}}^2 + \mathbf{X}'^2 = 1$. Here $\mathbf{X}(\sigma,t)$ denotes the spatial coordinates of points of the string at time t, the points being parametrized by the values of the parameter $\sigma \in [0, L]$, L being the total length of the loop, and the primes and dots indicate derivatives *w.r.t.* σ and t, respectively. In absence of any energy loss a closed loop will execute periodic motion with a fundamental period[28] $L/2$. Kibble and Turok[28] showed that any initially static[i.e., $\dot{\mathbf{X}}(\sigma,0) = 0$] loop (static in its c.m.frame) completely ***collapses*** after half a period of oscillation, i.e., $\mathbf{X}(\sigma, \frac{L}{4}) = \mathbf{X}(\sigma + \frac{L}{2}, \frac{L}{4})$, for all σ. (Here $t = 0$ is the time of formation of the loop). Complete collapse is also the fate of any initially non-static loop which has only single-frequency waves on it.[28] A special case of this is an exactly circular loop which completely collapses to a point. Of course, in reality, the string has a finite width ($\sim \mu^{-\frac{1}{2}}$), and when the radius of the loop becomes on the order of the width, the loop will probably turn into a massive particle which will then decay and produce energetic particles. (There is also a possibility[29] that the loop might form a black hole when it collapses; I will not consider this possibility here.) If a closed loop (not necessarily circular) completely collapses, the energy contained in the entire loop would be released[30] mostly in the form of massive gauge- and higgs bosons (of the underlying spontaneously broken gauge theory) as well as possible heavy fermions that could be coupled to the string-forming higgs field.[31] The decay products of these massive particles (the X-particles) would be present in today's universe in the form of UHE particles, provided the collapse occurred in not too early an epoch in the history of the universe. It is difficult, if not impossible, to have a precise estimate of the fraction of all loops that could be formed at any time in these collapsing configurations—all one can say is that this fraction is likely to be very small because these loops correspond to rather special configurations.

However, as a somewhat more general situation, one may consider closed loops which may not completely collapse but rather self-intersect, not just at one point, but at multiple (in principle, very large number of) points, i.e., there may be a large number of pairs (σ, σ') which satisfy the condition $\mathrm{X}(\sigma, t) = \mathrm{X}(\sigma', t)$ at some value of $t \in [0, \frac{L}{2}]$, $t = 0$ being the time of formation of the loop. In such a case, one loop will break up into a large number of smaller loops at once. Intersections of cosmic strings would be accompanied by particle production; because of the finite width of the string the two intersecting segments overlap at any point of intersection, and the underlying microphysical interactions of the fields 'constituting' the string would cause particle production at these overlapping regions near the points of intersection—the more the number of intersection points, the more will be the amount of energy released. Again it is hard to make a precise estimate of the amount of energy released in this process. In the following, we shall simply assume that a certain small average fraction f of the total energy of all primary loops formed at any time is released in the form of particles within one period of oscillation (specifically, at the end of half-period[28]) either due to a small fraction of loops completely collapsing and/or due to some loops breaking up into large number of smaller loops (due to multiple self-intersections) at once. We shall see that the observed UHE cosmic-ray flux gives an upper limit to this fraction f.

The loops which collapse or self-intersect themselves at time t are the ones that were formed at time $t_f = t(1+\alpha/4)^{-1}$. The X-particles are assumed to be produced instantaneously at the time of collapse or (multiple)self-intersections of the loops. So the rate of production of X-particles at time t, $dn_X(t)/dt$, where $n_X(t)$ is the number density, is given by the rate of loop formation at time t_f, Eq.1:

$$\frac{dn_X}{dt}(t) = f \cdot \left(\frac{dn_f}{dt_f}\right)_{t_f} \left(\frac{dt_f}{dt}\right) \left(\frac{a(t_f)}{a(t)}\right)^3 \frac{M_f}{m_X}. \tag{3}$$

In Eq.3, $a(t)$ is the scale factor of the universe, and m_X is the average energy of a single X-particle (we ignore here a possible spectrum of the emitted X-particles). We have also neglected the energy loss of the loops through gravitational radiation because the loops we are considering do not survive more than one oscillation period. We need to consider times t and t_f which are in the matter-dominated epoch only, because the particles produced at redshifts $z \gg O(1)$ essentially lose all energy during propagation through the cosmic medium and so they do not survive as UHE particles in the present epoch (see below). So, $a(t_f)/a(t) = (t_f/t)^{2/3}$. Eqs.(1)–(3) then give

$$\frac{dn_X(t)}{dt} = f\alpha\beta\mu m_X^{-1} t^{-3}. \tag{4}$$

4.4. Decay of X-Particles and Hadronization of the Decay Products

The X-particles released from the strings will decay, presumably into quarks and leptons. The quarks will hadronize producing jets of hadrons. The latter will be mostly pions, but a small fraction will be nucleons. The decays of the neutral pions in the hadronic jets will give rise to UHE gamma rays. The charged leptons in the decay products of the X-particles will also create electromagnetic cascades giving rise to a γ-ray background today. There will also be high-energy neutrinos resulting from the direct production of them by the decay of the X-particles, as well as from the decay of the charged pions in the hadronic jets. For the propagation times of our interest, most of the neutrons in the hadronic jets will ultimately end up as protons after β-decay. (Note, however, that for a mean lifetime of a neutron[32] $= 896\, sec$, a neutron having energy above $\sim 2.8 \times 10^{14} GeV$ will have a lifetime greater than the age of the universe, and hence will not decay.) The protons (as well as neutrons) lose energy as they propagate through the cosmic medium and appear today with degraded energy. The energy loss processes would include e^+e^- pair production and photopion production due to collision of protons with the background photons. These photoproduced secondary pions will also decay and give rise to a γ-ray background as well as a neutrino background, while the e^+e^- pairs will create electromagnetic cascades in the background photons and will also ultimately produce a γ-ray background. So there is a rich variety of physical processes which will have to be studied and their implications examined. In the following I shall only try to estimate the expected spectrum of the UHE primary ***protons*** resulting from the decay of the X-particles released from cosmic strings. The case of γ-rays and neutrinos will be discussed elsewhere.

Now, as already mentioned, each quark in the decay product of X will fragment and produce a jet of hadrons. To proceed further we will first have to know the fragmentation distribution function (FDF) of a jet, i.e., the number N_h of hadrons carrying a fraction x of the total energy in a jet. Unfortunately, the precise nature of the fragmentation process is not known and no 'first principle' derivation of any FDF is available. However, models yielding FDFs consistent with QCD expectations have been studied. Following Refs.10 and 11, we shall use here the following FDF formula that is consistent with the so-called "leading log QCD" behavior and seems to reproduce well the particle multiplicity growth as seen in GeV–TeV jets in colliders:

$$\frac{dN_h}{dx} \simeq 0.08\; exp\left(2.6\sqrt{ln(1/x)}\right)(1-x)^2\left(x\sqrt{ln(1/x)}\right)^{-1}. \tag{5}$$

In eq.(5), $x \equiv E/E_{jet}$, E being the energy of any hadron in the jet, and $E_{jet} \simeq m_X/2$ is the total energy in the jet (assuming that each X-particle decays into a quark and a lepton, each of which shares energy roughly equally, and each quark produces one hadronic jet). We assume that Eq. 5 provides a reasonable description of the hadronization process at the energies of our interest. The FDF described by Eq. 5 is displayed in Fig. 1 in which we also show a least-square-difference straight line fit to the $log(dN_h/dx)$ vs. $log\; x$ curve in the range $10^{-10} \leq x \leq 10^{-2}$, which gives a reasonable power-law approximation for the FDF, namely,

$$\frac{dN_h}{dx} \simeq 3.322\, x^{-1.324}, \quad 10^{-10} \leq x \leq 10^{-2}. \tag{6}$$

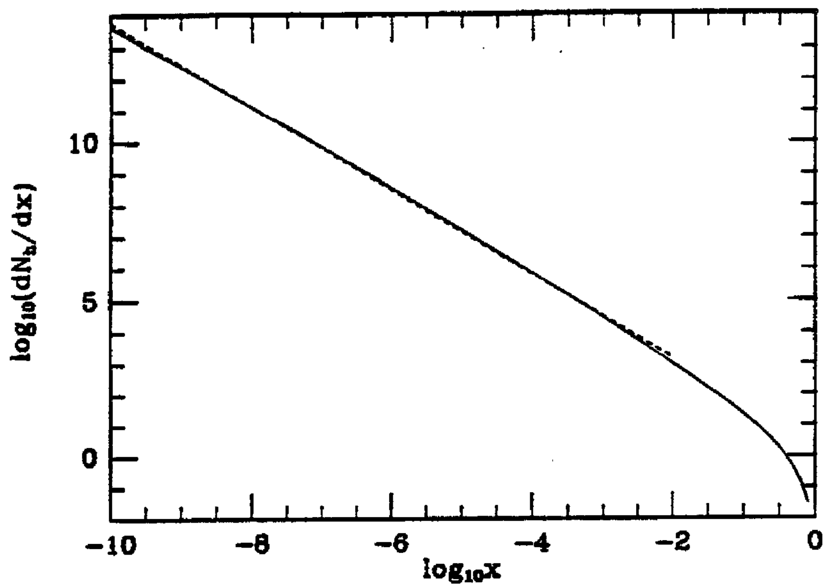

Fig. 1. The assumed jet Fragmentation Distribution Function (FDF) for quarks, i.e., the energy spectrum of the hadrons in a jet produced by a quark. The dashed curve is the least-square-difference straight line fit in the range $10^{-10} \leq x \leq 10^{-3}$ which gives a power-law, $\frac{dN_h}{dx} \simeq 3.322\, x^{-1.324}$ in the same range of x.

In the numerical calculations we use the full form of the FDF given in Eq. 5. The power-law form in Eq. 6 is, however, useful for approximate analytical calculations.

4.5. The Injection Spectrum of UHE Protons

As mentioned above, most of the hadrons in a jet will be pions, but a small fraction will be nucleons and antinucleons. It is not precisely known what this small fraction is— a reasonable estimate[11] is $\sim 3\%$. We shall assume that all the nucleons and antinucleons ultimately end up as protons and antiprotons after β-decay. This will clearly give us an upper limit to the proton flux because, as mentioned earlier, some neutrons and antineutrons, depending on their energy and propagation time, may have lifetime longer than the propagation time and so fail to decay before reaching us. In the following, protons and antiprotons will be collectively referred to simply as protons.

Now, let $\Phi(E_i, t_i)$ denote the injection spectrum of the protons, i.e., the number density of protons produced (i.e., injected) per unit energy interval at an injection energy E_i per unit time at an injection time t_i, due to the process we are considering. Then Eqs. 4 and 5 give, with $x = 2E_i/m_X$,

$$\Phi(E_i, t_i) \simeq 0.06(f\alpha\beta\eta)\left(\frac{M_{Pl}}{m_X}\right)^2 \left(\frac{dN_h}{dx}\right) t_i^{-3}, \tag{7}$$

where $\eta \equiv G\mu$ is the dimensionless cosmic string parameter, and we have assumed the nucleon content of a jet to be$\sim 3\%$. Thus Eqs. 6 and 7 yield a differential injection spectrum, $\Phi(E_i, t_i)$, that is approximately power law in E_i and t_i, namely, $\Phi(E_i, t_i) \propto E_i^{-1.324} t_i^{-3}$ for $\frac{m_X}{2} \cdot 10^{-10} \leq E_i \leq \frac{m_X}{2} \cdot 10^{-2}$. It is interesting to note that the basic form of the injection spectrum is determined by the fundamental microphysics of jet fragmentation in QCD and not by any external astrophysical parameters.

4.6. Energy Loss of Protons and the Flux in the Present Epoch

The injection spectrum obtained above will evolve due to the expansion of the universe as well as due to energy loss of the particles during their propagation through the cosmic medium. In order for a proton injected at a redshift z_i to appear today at any given energy E_0, it must have a definite injection energy $E_i \equiv E_i(E_0, z_i)$ such that $E_0 < E_i < \frac{1}{2}m_X$. Let $j(E_0)$ denote the expected flux, i.e., the number of protons per unit energy interval at an observed energy E_0, crossing per unit area per unit solid angle per unit time in the present epoch (t_0), due to the source $\Phi(E_i, t_i)$. Then assuming an isotropic distribution of the cosmic string loops in an Einstein-de Sitter "flat"($\Omega_0 = 1$) universe, we get[13]

$$j(E_0) = \frac{3}{8\pi} c t_0 \int_0^{z_{i,max}} dz_i (1+z_i)^{-5.5} \, \Phi(E_i, z_i) \left(\frac{dE_i(E_0, z_i)}{dE_0} \right)_{E_0}, \tag{8}$$

where z_i is the redshift corresponding to the injection time t_i, $t_i = t_0(1+z_i)^{-1.5}$, and the upper limit $z_{i,max}(E_0)$ on the integral is defined such that

$$E_i(E_0, z_{i,max}(E_0)) = m_X/2. \tag{9}$$

The dominant energy loss of UHE protons propagating through the cosmic medium at an epoch of redshift z is due to the following processes[33]: (i)Cosmological redshift due to expansion of the universe, (ii)e^+e^--pair production off the background photons($p + \gamma \to p + e^+ + e^-$), and (iii)photopion production off the background photons($p + \gamma \to \pi + N$). Thus one can write[35]

$$\frac{1}{E}\frac{dE}{dz} = (1+z)^{-1} + H_0^{-1}(1+z)^{\frac{1}{2}} \left[\beta_{0,pair}\left((1+z)E\right) + \beta_{0,pion}\left((1+z)E\right) \right], \tag{10}$$

where the function $\beta_0(E) \equiv -\frac{1}{E}\left(\frac{dE}{dt}\right)_{z=0}$ denotes the energy-loss rate (divided by the energy) *in the present epoch* ($z = 0$) of a proton of energy E; the subscripts "pair" and "pion" refer respectively to the processes (ii) and (iii) mentioned above. The first term on the right hand side of Eq. 10 corresponds to energy loss of the particles due to expansion of the universe. In writing Eq. 10, we have used the fact that the energy loss due to pair- or photopion-production at any epoch with redshift z is related to that in the present epoch through the relation $\beta(E, z) = (1+z)^3 \beta((1+z)E)$ which follows from the fact that the number density of background photons was higher by a factor of $(1+z)^3$ and energy of each photon higher by a factor of $(1+z)$ at the epoch with redshift z compared to the respective values of these quantities in the present epoch.

The energy-loss function $\beta_0(E)$ has been calculated by several authors. For a nice summary see Fig. 1 of Ref. 35 whose results we shall use below. For $E \lesssim 6 \times 10^{19} eV$, $\beta_{0,pair}$ dominates over $\beta_{0,pion}$; $\beta_{0,pair}^{-1}(E)$ decreases from $\sim 10^{11} yr$ at $E \simeq 10^{18} eV$ to $\sim 7.8 \times 10^9 yr$ at $E \simeq 4.6 \times 10^{18} eV$. For $5 \times 10^{18} eV \lesssim E \lesssim 6 \times 10^{19} eV$, $\beta_{0,pair}^{-1}(E)$ has a weak energy dependence remaining roughly constant at $\sim 5 \times 10^9 yr$. At $E \gtrsim 6 \times 10^{19} eV$, $\beta_{0,pion}$ becomes dominant and it rises very steeply with increasing E; $\beta_{0,pion}^{-1}$ decreases from $\sim 4.7 \times 10^9 yr$ at $E \simeq 6 \times 10^{19} eV$ to $\sim 7.9 \times 10^7 yr$ at $E \simeq 2 \times 10^{20} eV$. Above $\sim 10^{21} eV$, $\beta_{0,pion}^{-1}$ reaches a roughly constant value at $\sim 3.9 \times 10^7 yr$. The sharp fall-off of the time scale of energy loss through photopion production for $E \gtrsim 6 \times 10^{19} eV$ is the basis of the well-known Greisen-Zatsepin-Kuz'min (GZK)[36]- prediction of an expected onset of a cutoff of the UHE cosmic ray proton spectrum at $E \gtrsim 6 \times 10^{19} eV$, provided the sources are extragalactic.

Now, knowing $\beta_{0,pair}(E)$ and $\beta_{0,pion}(E)$, our task is to first solve Eq. 10 numerically to find the values of E_i for given values of z_i and E_0. The derivatives (dE_i/dE_0) can also be evaluated numerically. The injection spectrum is then calculated from Eqs. 5 and 7, and the numerical evaluation of the z_i-integral in Eq. 8 gives the required flux. The value of $z_{i,max}$ defined by Eq. 9 increases as E_0 decreases. But for all values of E_0 of our interest, $z_{i,max}$ remains below ~ 2. Actually, the contributions to the flux from those values of z_i for which $E_i(E_0, z_i) \gg 6 \times 10^{19} eV$, fall sharply as z_i increases. This is because of the fact that for these values of z_i the E_i's are in the photopion-energy-loss regime, and in this regime the *rate of energy loss* itself rises sharply with energy so that energy E_i rises very sharply indeed (exponentially or faster) as z_i increases. Since the injection spectrum $\Phi(E_i, z_i)$ falls (roughly as a power law $\propto E_i^{-1.324}$, see Eqs. 6 and 7) with E_i, and since E_i itself rises exponentially or faster with z_i, it follows that the rapid decrease of the injection spectrum $\Phi(E_i, z_i)$ with increasing z_i due to energy loss of the particles dominates over the power-law rise of $\Phi(E_i, z_i)$ with increasing z_i due to higher number density of the cosmic string loops at higher redshifts (see Eq. 7). This in fact ensures that the z_i-integral in Eq. 8 converges fast.

5. Results and Discussions

The results of the calculations described above are shown in Figs. 2 and 3 for one particular value of $m_X = 10^{15} GeV$ (GUT scale). In Fig. 2 is shown the flux $j(E_0)$, and in Fig. 3 the quantity $E_0^3 j(E_0)$ as functions of the observed energy E_0.

The observational data on the UHE cosmic-ray flux differ among the various experimental groups. For definiteness we consider here the best-fit power-law result for the UHE cosmic-ray flux given by the "Fly's Eye" group,[37] which is represented by the dashed curves in Figs. 2 and 3. By requiring that the flux due to the process under consideration remain below the observed flux at all energies, we find an upper limit on the value of the product $f\alpha\beta\eta$, namely, $f\alpha\beta\eta \leq 1.7 \times 10^{-9}$; the solid curves in Figs. 2 and 3 represent the 'upper-limit' results i.e., for $f\alpha\beta\eta = 1.7 \times 10^{-9}$. Then with $\alpha\beta = 0.57$ (determined by numerical simulations, see sec. 4.2),

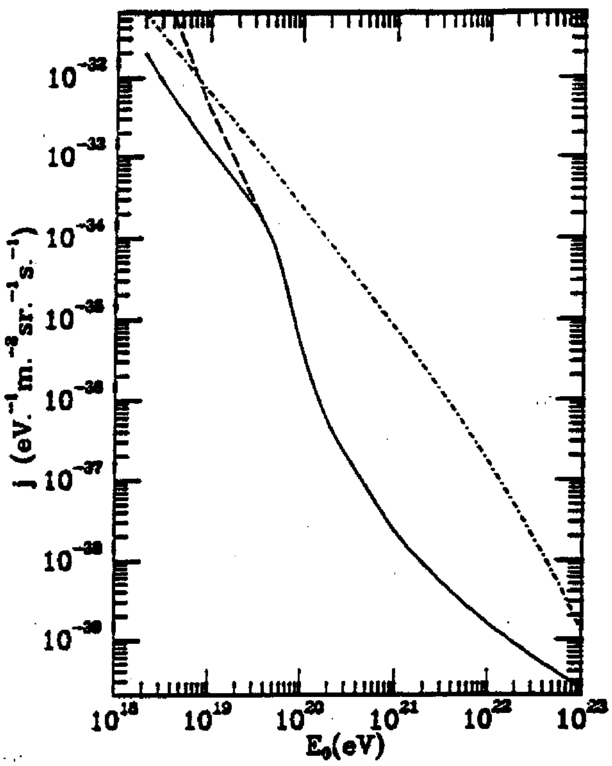

Figure 2. The estimated UHE proton flux $j(E_o)$ for the upper limit value of the multiplicative constant $f\alpha\beta\eta = 1.7 \times 10^{-9}$ (solid curve), and $m_X = 10^{15}GeV$. The dashed curve represents the best-fit power-law result for the observed UHE cosmic ray spectrum given by the "Fly's Eye" group.[37] The dash-dotted curve represents the flux that would be obtained (again, with $f\alpha\beta\eta = 1.7 \times 10^{-9}$ and $m_X = 10^{15}GeV$) if the protons did not suffer any energy loss *except* that due to the expansion of the universe.

the above upper limit on $f\alpha\beta\eta$ gives $f\eta \leq 2.98 \times 10^{-9}$. Now, recall that η, the dimensionless mass per unit length of the string, is determined by the temperature in the early universe at which the cosmic-string-forming phase transition occurs — the lower the phase transition temperature, the 'lighter' is the string. There is an independent upper limit on the value of η — it comes from the consideration of the null results for the expected pulsar timing variation[38] due to the stochastic gravitational wave background[39] that would be created by stably oscillating non-self-intersecting cosmic string loops. The limit given by Bouchet and Bennett[40] from their numerical simulations together with the results from the millisecond pulsar timing experiment,[41] is $\eta \leq 4 \times 10^{-6}$. If we take here the upper-limit value, namely, $\eta = 4 \times 10^{-6}$, which is appropriate for GUT-scale cosmic strings, then we get $f \leq 7.46 \times 10^{-4}$. Clearly, very light strings ($\eta \ll 10^{-6}$), if they exist, contribute little to the UHE proton flux today, and so for very light cosmic strings not much useful constraint on the parameter f can be obtained from the observed UHE cosmic

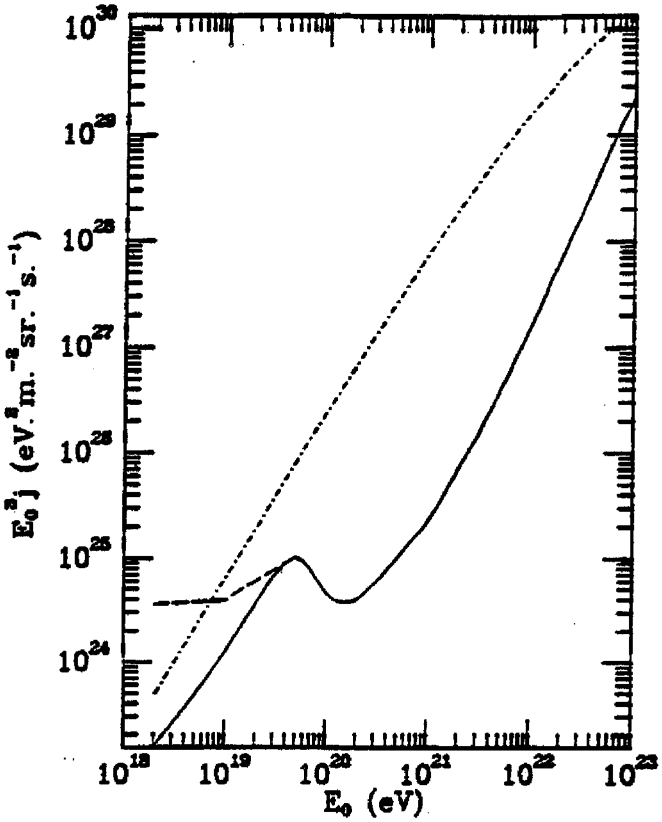

Figure 3. The same as in Fig. 2 except that the quantity $E_0^3 j(E_0)$ is plotted instead of $j(E_0)$.

ray flux. However, as the results obtained above show, observational results on UHE cosmic rays do imply that a large fraction of *GUT-scale* cosmic string loops must be in non-self-intersecting and non-collapsing configurations. In the following, by cosmic strings, we refer to GUT-scale cosmic strings only.

The solid curves in Figs. 2 and 3 are obtained when we fully include the energy loss of the protons in collision with the cosmic background radiation. For comparison, I show in these same Figures the results — represented by the dash-dotted curves — that would be obtained (again, with $f\alpha\beta\eta = 1.7 \times 10^{-9}$ and $m_X = 10^{15} GeV$), if the particles did not suffer any energy loss *except* the loss due to the expansion of the universe. In other words, the dash-dotted curves in Figs. 2 and 3 essentially represent the 'redshifted' *injection* spectrum integrated over all redshifts. Note that in this case, the maximum possible injection redshift $z_{i,max}(E_0)$ for a given value of the observed energy E_0 (see Eq. 9) is given by $(1 + z_{i,max}(E_0)) = m_X/2$, and so for $E_0 < (m_X/10^{15} GeV) \times 3.176 \times 10^{19} eV$, we have $z_{i,max} > z_{eq}$. Therefore, Eqs. 4, 7, and 8, which are valid only for $t_i > t_{eq}$, or $z_i < z_{eq}$ (i.e., in the matter-dominated epoch only), have to be modified appropriately to their forms valid in the radiation dominated epoch, when calculating the contributions coming from redshifts $z_i > z_{eq}$ — this has been done in obtaining the dash-dotted curves in Figs. 2 and 3. From Figs. 2 and 3 we see how, for $E_0 \gtrsim 6 \times 10^{19} eV$, the energy loss of the protons due to photopion production off the (microwave) background photons drastically modifies the redshifted injection spectrum. The well known expected feature[34,35], the so-called "pile up" or "bump" at $E_0 \sim 6 \times 10^{19} eV$, due to 'photopion energy loss', is clearly visible in Fig. 3. But perhaps the most interesting feature of the spectrum represented by the solid curves in Figs. 2 and 3 is that there is really *no complete* GZK-cutoff associated with the photopion energy loss; instead $E_0^3 j(E_0)$ (Fig. 3) only takes a "dip" after the "pile up" and then rises again as E_0 increases further, before the final cutoff (not shown) at $E_0 = m_X/2$. This sequence of structures in the spectrum is reflected in Fig. 2 as first a steepening of $j(E_0)$ beginning at $E_0 \sim 6 \times 10^{19} eV$, then a progressive *flattening* beginning at $E_0 \sim 10^{20} eV$, until a final cutoff starts near $E_0 = m_X/2$. The 'recovery' of the spectrum after the GZK-cutoff is partly due to progressive decrease of the slope of the curve for $\beta_{0,pion}$ beginning at $E_0 \sim 10^{20} eV$, and more importantly, due to the continued — albeit diminishing — strength of the injection spectrum at energies beyond this value.

From Figs. 2 and 3 it is clear that the spectrum of UHE protons resulting from the particular cosmic-string-induced process that we have discussed above is quite different from the spectrum of UHE cosmic rays in the observed energy range, and it is unlikely that this process gives any significant contribution to the observed UHE cosmic rays at energies below $\sim 3 \times 10^{19} eV$. However, above this energy, the process *can* indeed be a significant contributor to the UHE proton flux today. Observationally, the Fly's Eye group[37] essentially sees no events above the energy $7 \times 10^{19} eV$(the "cutoff"), whereas the Haverah Park experiment[42], for example, reports events at $10^{20} eV$ and higher. I am in no position to shed any light on this observational situation.[43] However, as is clear from the above discussions, the particular scenario we have discussed above, and possibly other processes involving topological defects as well, may give rise to energetic events, albeit at a very small rate, beyond the GZK 'cutoff' in a natural way. Indeed, from the calculations discussed above, one can make a crude estimate of the expected *integral flux* of UHE protons above any given energy, due to the particular process we have considered above. We get, for $f\alpha\beta\eta = 1.7 \times 10^{-9}$ (see above), an integral flux $J(E_0 > 6 \times 10^{19} eV) \simeq 9.1 \times 10^{-16} m^{-2}.s^{-1}.sr^{-1}$, which gives an event rate $N(E_0 > 6 \times 10^{19} eV) \simeq 9 \times 10^{-2} yr^{-1}.Km^{-2}$. Note that this number represents an upper limit for the process under consideration— depending on the value of the fraction f, the number could be smaller. Nevertheless, one hopes that, if f is not too small, the energetic events at energies significantly above $\sim 6 \times 10^{19} eV$, due to the cosmic-string-induced process discussed here, may be detectable in some of the planned future Extended Air Shower experiments with larger area coverage than currently available. The detection of cosmic-ray events above an energy of, say $10^{22} eV$, will almost certainly be one of the few 'signatures' of GUT and, more specifically, of the existence of topological defects. For, as mentioned in sec. 2, it is hard to think of an astrophysical mechanism that could accelerate particles to such high energies.

What would be a characteristic feature of cosmic ray events purely attributable to the kind of topological-defect-induced process discussed above? One distinguishing feature may be that the hadronic component of the events will primarily consist of protons and perhaps some neutrons too, but *no* "heavies" such as Fe nuclei — certainly there is no way quarks coming from the decay of the X-particles released by the topological defects can fragment directly into nuclei! Another point to be noted is that there should be nucleons ***as well as antinucleons*** in the fragmentation product of the quarks, and so the primary composition will have $\bar{p}$'s, $\bar{n}$'s in addition to the p's and n's. However, at the energies we are concerned with, the extensive air showers created by p's and those created by $\bar{p}$'s would, for all practical purposes, be identical and hence indistinguishable.

Finally, a few words about the assumption of isotropic distribution of the cosmic string loops in the calculations described above. Clearly, it is not a good approximation to make for very small redshifts for which the number density of loops is small and the mean separation between loops is large. Roughly, the assumption breaks down for redshifts for which the distance to the loops is less than the mean separation between the loops. The number density of the primary loops formed at redshift z is, from Eq. 1, $n(z) = \frac{7}{3}\beta(1+\frac{\alpha}{4})(ct_0)^{-3}(1+z)^{4.5}$, so that the mean separation between these loops is $n^{-1/3}(z) = \left(\frac{7}{3}\beta(1+\frac{\alpha}{4})\right)^{-1/3} ct_0(1+z)^{-1.5}$. The coordinate distance $d(z)$ to a loop at redshift z in an $\Omega_0 = 1$ universe is given by $d(z) = \frac{2c}{H_0}\left(1-(1+z)^{-0.5}\right)$. One can then check that for $\alpha\beta = 0.57$, $\alpha = 50\eta$ (see sec. 4.2) and $\eta = 4\times 10^{-6}$ (see sec. 5), the condition $n^{-1/3}(z) < d(z)$ breaks down for $z \lesssim 0.033$. This implies that the contribution to the integral on the right-hand-side of Eq. 8 coming from the loops with $z_i \gtrsim 0.033$ will be responsible for an isotropic diffuse component of the flux, while the total flux will have an anisotropic component due to isolated relatively nearby loops at $z_i < 0.033$. Since the dominant contribution to the highest-energy end of the spectrum comes from the relatively nearby loops, we expect that the anisotropy will be prominent in the highest energy region.

The typical size of the primary loops formed at redshift z is $R(z) = \alpha ct(z) = 0.53(1+z)^{-1.5}Mpc$ for $\alpha = 2\times 10^{-4}$ (see above). Thus, for $z = 1$, for example, the mean separation between the newly formed primary loops is $\sim 50\ Mpc$, the typical size of a loop is $\sim 0.2\ Mpc$, and the distance from us to a loop is $\sim 2340.8\ Mpc$. So, the loops at these 'large' redshifts may be treated as point objects. However, for very small redshifts, the extended nature of the loops may be important.

6. Summary and Conclusions

In summary, then, the possibility that topological defects formed at a GUT-scale phase transition may provide us with a fundamental mechanism of production of UHE particles extending in energy to $\sim 10^{24}eV$, without the need for any acceleration mechanism, is very exciting. In this talk I have discussed the general idea behind the scenario of production of UHE particles from topological defects, and also discussed a specific process which involves collapse or multiple self-intersections of closed cosmic string loops. A general property of topological defect-induced cosmic rays is that the injection spectrum of the particles is determined by the microphysics of hadronization and jet fragmentation in QCD and not by any external astrophysical parameters. I have shown how the observed UHE cosmic ray spectrum gives us an upper limit to the average fraction f of the total energy of all primary cosmic string loops, that can be released in the form of energetic particles through the process of collapse or multiple self-intersection of some of these loops. A characteristic feature of the final spectrum of UHE protons is the absence of a complete GZK[36] 'cutoff' at $E_0 \sim 6 \times 10^{19} eV$ associated with energy loss of UHE protons due to photopion production off the microwave background radiation. The 'recovery' of the UHE proton spectrum after the GZK-'cutoff' is due to the fact that the injection spectrum continues to an energy $\sim 10^{24} eV$. The specific process we have considered does not give any significant contribution to the observed UHE cosmic ray proton spectrum at energies below $\sim 3 \times 10^{19} eV$. However, above this energy the process can, in principle, give significant contribution to the observed flux. In particular, the integral flux of protons above the energy $6 \times 10^{19} eV$ may be large enough to be detectable by the planned future UHE cosmic ray experiments with large area coverage. A specific prediction of the theory of topological defect-induced cosmic rays is that the hadronic component of UHE cosmic rays will consist of 'fundamental' particles, namely, p's, n's and their antiparticles, but *no* nuclei such as α's or Fe nuclei.

Clearly, much work remains to be done on this subject. In this talk I have discussed only one particular process of UHE particle production from topological defects. There are other processes which are now being studied. Moreover, in this talk, I have concentrated on the UHE *protons* only. As mentioned earlier, there would also be UHE neutrinos and gamma rays produced from topological defects. These have important implications which, however, are outside the scope of this talk, and will be discussed elsewhere.

Let me finally conclude by mentioning again that discovery of cosmic ray particles with energies in excess of say $\sim 10^{22} eV$ may provide us with one of the few 'signatures' of GUT in general and existence of topological defects in particular.

Acknowledgements

I thank the organizers of the Symposium for inviting me to give this talk, and for their warm hospitality. I have enjoyed stimulating discussions with a number of participants, in particular, with M. Teshima, A. Wolfendale and G. Yodh. It is a pleasure to thank D. Schramm for discussions, encouragement and support. I would also like to thank N. C. Rana for his collaboration in a part of the work discussed above. This work was supported by NSF grant #87-15412 to the University of Chicago, and in part by DOE and the NASA (grant NAGW-1340) at Fermilab.

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